



- Integration is inverse process of differentiation.
- Integration is process of finding a function, whose d.c in known.

$$\int f'(x) dx = f(x) + c$$

BASIC FORMULAE:

Derivatives

1. $\frac{d}{dx} \left(\frac{x^{n+1}}{n+1} \right) = x^n$
2. $\frac{d}{dx} (\sin x) = \cos x$
3. $\frac{d}{dx} (-\cos x) = \sin x$
4. $\frac{d}{dx} (\tan x) = \sec^2 x$
5. $\frac{d}{dx} (-\cot x) = \operatorname{cosec}^2 x$
6. $\frac{d}{dx} (\sec x) = \sec x \cdot \tan x$
7. $\frac{d}{dx} (-\operatorname{cosec} x) = \operatorname{cosec} x \cdot \cot x$
8. $\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
9. $\frac{d}{dx} (-\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
10. $\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$
11. $\frac{d}{dx} (-\cot^{-1} x) = \frac{1}{1+x^2}$
12. $\frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
13. $\frac{d}{dx} (-\operatorname{cosec}^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
14. $\frac{d(e^x)}{dx} = e^x$
15. $\frac{d}{dx} (\log|x|) = \frac{1}{x}$
16. $\frac{d}{dx} (a^x) = a^x \log a$

Integrals(Anti derivatives)

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
2. $\int \cos x dx = \sin x + c$
3. $\int \sin x dx = -\cos x + c$
4. $\int \sec^2 x dx = \tan x + c$
5. $\int \operatorname{cosec}^2 x dx = -\cot x + c$
6. $\int \sec x \cdot \tan x dx = \sec x + c$
7. $\int \operatorname{cosec} x \cdot \cot x dx = -\operatorname{cosec} x + c$
8. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$
9. $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + c$
10. $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$
11. $\int \frac{dx}{1+x^2} = -\cot^{-1} x + c$
12. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$
13. $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + c$
14. $\int e^x dx = e^x + c$
15. $\int \frac{1}{x} dx = \log|x| + c$
16. $\int a^x dx = \frac{a^x}{\log a} + c$

Integration of some other function:

1. $\int \tan x \, dx = \log|\sec x| + c$
2. $\int \cot x \, dx = \log|\sin x| + c$
3. $\int \sec x \, dx = \log|\sec x + \tan x| + c$
4. $\int \cosec x \, dx = \log|\cosec x - \cot x| + c$
5. $\int \frac{dx}{x^2-1} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$
6. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$
7. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
8. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x + \sqrt{x^2-a^2}| + c$
9. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$
10. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x + \sqrt{x^2+a^2}| + c$

Method to find the Integration:

(1) Integration by parts:

$$\int u \cdot v \, dx = u \int v \, dx - \int \left(\frac{du}{dx} \int v \, dx \right) dx$$

Note: we take u that function whose first letter first come in “ILATE”

I – Inverse Trigonometric function.

L – Logarithmic function.

A – Algebraic function.

T – Trigonometric function.

E – Exponential function.

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$$

(2) Integration by substitution:

To Evaluate $\int \{f(x) \cdot \varphi'(x)\} dx$

Put $\varphi(x) = t$ and $\varphi'(x)dx = dt$

Where $\varphi'(x)$ is the differential coefficient of $\varphi(x)$ with respect to x

(3) Integration by Partial fraction:

Form of Rational fraction

$$1. \frac{px+q}{(x-a)(x-b)} \quad a \neq b$$

$$2. \frac{px+q}{(x-a)^2}$$

Form of Partial fraction

$$\frac{A}{(x-a)} + \frac{B}{(x-b)}$$

$$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$$

$$3. \frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$$

$$4. \frac{px+qx+r}{(x-a)^2(x-b)}$$

$$5. \frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$$

$$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$$

$$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-c)}$$

$$\frac{A}{(x-a)} + \frac{Bx+c}{(x^2+bx+c)}$$

where $(x^2 + bx + c)$ can not be factorize

Some Properties of Definite Integrals:

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$
2. $\int_a^b f(x)dx = - \int_b^a f(x)dx$
3. $\int_a^a f(x)dx = 0$
4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
5. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
6. $\int_0^b f(x)dx = \int_0^b f(a-x)dx$
7. $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
8. $\int_0^{2a} f(x)dx = \begin{cases} 2 \int_0^a f(x)dx, & \text{if } 2a-x = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$
9. $\int_{-a}^a f(x)dx = \begin{cases} 2 \int_0^a f(x)dx, & \text{if } f \text{ is even function } f(-x) = f(x) \\ 0, & \text{if } f \text{ is odd function } f(-x) = -f(x) \end{cases}$

Important formula

1. $2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$
2. $2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$
3. $2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$
4. $2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$
5. $\sin 3A = 3\sin A - 4\sin^3 A$
6. $\cos 3A = 4\cos^3 A - 3\cos A$
7. $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$
8. $\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$
9. $\sin 2A = 2\sin A \cdot \cos A = \frac{2\tan A}{1 + \tan^2 A}$